

## AP Calculus AB Summer Assignment

Name: \_\_\_\_\_

**Due Date:** *The last class day of the first week of school.*

The purpose of this assignment is to have you practice the mathematical skills necessary to be successful in AP Calculus AB. Most of the skills covered in this packet are skills from Algebra 2 and Pre-Calculus, some are skills from an introduction to the Calculus. In particular, it will be very important for you to know your trig-identities prior to the first day of school. If you need to, you may use reference materials to assist you and refresh your memory (old notes, textbooks, online resources, etc.). I highly recommend you reference Khan Academy as it is an excellent resource to you which we will use throughout the year. While the graphing calculators will be used in class, there are *no calculators allowed* on this packet. You should be able to do everything without a calculator.

AP Calculus AB is a fast paced course that is taught at the college level. There is a lot of material in the curriculum that must be covered before the AP exam in May. Therefore, we cannot spend a lot of class time re-teaching prerequisite skills. This is why you have this packet. Spend some time with it and make sure you are clear on everything covered in the packet so that you will be successful in Calculus. Of course, you are always encouraged to seek help from your teacher if necessary.

**Your assignment is to do at least the odd-numbered problems. Do more if you feel you need the practice.**

This assignment will be collected and graded the last class day of the first week of school. Be sure to show all appropriate work to support your answers. In addition, there may be a quiz on this material when we return to school.

***Good Luck!***

***Mr. Tavoletto***

Name: \_\_\_\_\_

Show all work - no credit will be awarded for answers missing appropriate work.  
No calculators!

### Section I: Algebra Review

Identify the following statements as true or false.

1.  $\frac{x+y}{2} = \frac{x}{2} + \frac{y}{2}$  \_\_\_\_\_

2.  $\frac{1}{p+q} = \frac{1}{p} + \frac{1}{q}$  \_\_\_\_\_

3.  $\frac{2k}{2x+h} = \frac{k}{x+h}$  \_\_\_\_\_

4.  $3 \cdot \frac{a}{b} = \frac{3a}{b}$  \_\_\_\_\_

5.  $3 \cdot \frac{a+b}{c} = \frac{3a+b}{c}$  \_\_\_\_\_

6.  $\sqrt{a^2 + b^2} = a + b$  \_\_\_\_\_

Identify the following statements as true or false over the set of real numbers. Give a counter example for any false statement.

7.  $x^3 + 1 > x^3$  \_\_\_\_\_

8.  $x^3 + x > x^3$  \_\_\_\_\_

9.  $x^2 \geq 0$  \_\_\_\_\_

10.  $x^2 \geq x$  \_\_\_\_\_

11.  $2x \geq x$  \_\_\_\_\_

12.  $\sqrt{x} \geq 0$  \_\_\_\_\_

13.  $-x \leq 0$  \_\_\_\_\_

14.  $\frac{1}{x} \leq x$  \_\_\_\_\_

15.  $x \leq |x|$  \_\_\_\_\_

16. Solve  $xy' + y + 1 = y'$  for  $y'$ .

17. Solve  $\ln y = kt$  for  $y$ .

16. \_\_\_\_\_

18. Factor:  $y^3 + 27$

19. Factor:  $x^2(x-1) - 4(x-1)$

17. \_\_\_\_\_

18. \_\_\_\_\_

19. \_\_\_\_\_

Simplify each expression.

20.  $\frac{(x^2)^3 x}{x^7}$  \_\_\_\_\_

21.  $\sqrt{x} \cdot \sqrt[3]{x} \cdot x^{\frac{1}{6}}$  \_\_\_\_\_

22.  $\frac{5(x+h)^3 - 5x^3}{h}$  \_\_\_\_\_

23.  $\frac{3(x+h)^2 - 3x^2}{h}$  \_\_\_\_\_

$$24. \frac{\frac{x^2 - 1}{x}}{\frac{x+1}{x^3}} \underline{\hspace{2cm}}$$

$$25. \frac{\frac{1}{x} + \frac{4}{x^2}}{3 - \frac{1}{x}} \underline{\hspace{2cm}}$$

$$26. \frac{\frac{a}{2x+h} - \frac{a}{2x}}{h} \underline{\hspace{2cm}}$$

$$27. \frac{1}{1-2a} - \frac{2}{1+2a} + \frac{6a+2}{4a^2-1} \underline{\hspace{2cm}}$$

*Simplify, using factoring of binomial expressions. Leave answers in factored form.*

**Example:**

$$\begin{aligned} \frac{(x+1)^3(4x-9) - (16x+9)(x+1)^2}{(x-6)(x+1)} &= \frac{(x+1)^2[(x+1)(4x-9) - (16x+9)]}{(x-6)(x+1)} \\ &= \frac{(x+1)^2(4x^2 - 5x - 9 - 16x - 9)}{(x-6)(x+1)} \\ &= \frac{(x+1)^2(4x^2 - 21x - 18)}{(x-6)(x+1)} \\ &= \frac{(x+1)^2(4x+3)(x-6)}{(x-6)(x+1)} \\ &= (x+1)(4x+3) \end{aligned}$$

$$28. (x-1)^3(2x-3) - (2x+12)(x-1)^2 \underline{\hspace{2cm}}$$

$$29. \frac{(x-1)^2(3x-1) - 2(x-1) \cdot 3}{(x-1)^4} \underline{\hspace{2cm}}$$

$$30. \frac{(x-1)^3(2x-3) - (4x-1)(x-1)^2}{(x-1)^2(2x-1)} \underline{\hspace{2cm}}$$

Simplify by rationalizing the numerator.

Example:

$$\frac{\sqrt{x+4}-2}{x} = \frac{\sqrt{x+4}-2}{x} \cdot \frac{\sqrt{x+4}+2}{\sqrt{x+4}+2} = \frac{x+4-4}{x(\sqrt{x+4}+2)} = \frac{x}{x(\sqrt{x+4}+2)} = \frac{1}{\sqrt{x+4}+2}$$

31.  $\frac{\sqrt{x+9}-3}{x}$  \_\_\_\_\_

32.  $\frac{\sqrt{x+h}-\sqrt{x}}{h}$  \_\_\_\_\_

Solve each equation or inequality for  $x$  over the set of real numbers.

33.  $2x^4 + 3x^3 - 2x^2 = 0$  \_\_\_\_\_

34.  $\frac{2x-7}{x+1} = \frac{2x}{x+4}$  \_\_\_\_\_

35.  $\frac{3x+5}{(x-1)(x^4+7)} = 0$  \_\_\_\_\_

36.  $\sqrt{x^2-9} = x-1$  \_\_\_\_\_

37.  $|2x-3| = 14$  \_\_\_\_\_

38.  $x^2 - 2x - 8 < 0$  \_\_\_\_\_

Solve each of the systems.

39.  $x + y = 8$  \_\_\_\_\_  
 $2x - y = 7$

40.  $y = x^2 - 3x$  \_\_\_\_\_  
 $y = 2x - 6$

## Section II: Pre-Calculus Review

Use your knowledge of the unit circle to evaluate each of the following. Leave your answers in radical form.

41.  $\sin(30^\circ)$  \_\_\_\_\_

42.  $\cos \frac{2\pi}{3}$  \_\_\_\_\_

43.  $\tan 45^\circ$  \_\_\_\_\_

44.  $\sin\left(-\frac{\pi}{6}\right)$  \_\_\_\_\_

45.  $\tan \pi$  \_\_\_\_\_

46.  $\csc \frac{5\pi}{6}$  \_\_\_\_\_

47.  $\cos(90^\circ)$  \_\_\_\_\_

48.  $\cos \frac{3\pi}{4}$  \_\_\_\_\_

49.  $\tan \frac{\pi}{6}$  \_\_\_\_\_

50.  $\cos^{-1}\left(\frac{1}{2}\right)$  \_\_\_\_\_

51.  $\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$  \_\_\_\_\_

52.  $\tan^{-1}(1)$  \_\_\_\_\_

Solve each trigonometric equation for  $0 \leq x \leq 2\pi$ .

53.  $\sin x = \frac{\sqrt{3}}{2}$  \_\_\_\_\_

54.  $\tan^2 x = 1$  \_\_\_\_\_

55.  $\cos \frac{x}{2} = \frac{\sqrt{2}}{2}$  \_\_\_\_\_

56.  $2\sin^2 x + \sin x - 1 = 0$  \_\_\_\_\_

For each trigonometric function identify the amplitude and any horizontal or vertical shifts from the basic function.

57.  $y = \frac{1}{2}\cos \frac{x}{2} - 3$  amplitude: \_\_\_\_\_ period: \_\_\_\_\_ vertical shift: \_\_\_\_\_

58.  $y = 2\sin(2x - \pi)$  amplitude: \_\_\_\_\_ period: \_\_\_\_\_ horizontal shift: \_\_\_\_\_

59.  $y = \tan 3x$  period: \_\_\_\_\_

Solve each exponential or logarithmic equation.

60.  $5^x = 125$  \_\_\_\_\_

61.  $8^{x+1} = 16^x$  \_\_\_\_\_

62.  $81^{\frac{3}{4}} = x$  \_\_\_\_\_

63.  $8^{\frac{-2}{3}} = x$  \_\_\_\_\_

64.  $\log_2 32 = x$  \_\_\_\_\_

65.  $\log_x \frac{1}{9} = -2$  \_\_\_\_\_

66.  $\log_4 x = 3$  \_\_\_\_\_

67.  $\log_3(x+7) = \log_3(2x-1)$  \_\_\_\_\_

Expand each of the following using the laws of logs.

68.  $\log_3 5x^2$  \_\_\_\_\_

69.  $\ln \frac{5x}{y^2}$  \_\_\_\_\_

Complete each of the following using trigonometric identities and formulas.

70.  $\sin\left(\frac{\pi}{2} - x\right) =$  \_\_\_\_\_

71.  $\sin^2 x + \cos^2 x =$  \_\_\_\_\_

72.  $\sin 2u =$  \_\_\_\_\_

73.  $\tan x =$  \_\_\_\_\_

74.  $1 + \cot^2 x =$  \_\_\_\_\_

75.  $1 - \cos^2 x =$  \_\_\_\_\_

76. A right triangle has a base of 5 and a hypotenuse of 7. Find the height.

### Section III: Graphing Review

Sketch the following functions. State the domain and range of each. Draw and label your own axes.

77.  $f(x) = x$

78.  $f(x) = x^2$

79.  $f(x) = x^3$

80.  $f(x) = |x|$

81.  $f(x) = [x]$  (Greatest integer function)

82.  $f(x) = \frac{1}{x}$

83.  $f(x) = \sqrt{x}$

84.  $f(x) = e^x$

85.  $f(x) = \ln x$

86.  $f(x) = \sqrt{9 - x^2}$

87.  $f(x) = \sin x$

88.  $f(x) = \cos x$

89.  $f(x) = \tan x$

90.  $f(x) = \csc x$

91.  $f(x) = \sec x$

92.  $f(x) = \cot x$

## Section IV: Intro to Calculus Review - Limits and Derivatives

Evaluate each limit.

$$93. \lim_{x \rightarrow 0} 3 \cos 2x + 2 \text{ _____}$$

$$94. \lim_{x \rightarrow 3} \frac{x^2 - 4x + 3}{x - 3} \text{ _____}$$

$$95. \lim_{x \rightarrow 4} \frac{\sqrt{x} - 2}{x - 4} \text{ _____}$$

Use the limit definition of derivative to find the derivative of:

$$96. 4x^2 + 3x - 5$$

Use the derivative rules to find the derivative of each function.

$$97. f(x) = 3x^2 - 5x + 11 \text{ _____}$$

$$98. f(x) = 5x^3(x^4 - 3x^2) \text{ _____}$$